

Mechanical Vibration

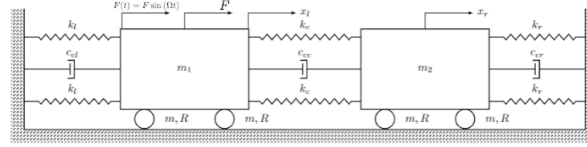
Multi Degree of Freedom DAMPED SYSTEM

Forced Damped Trolleys With Springs Report with 2DOF for $m_1 = m$,
 $m_2 = m$, $c_r = k_c \lambda$, $c_l = k_c \lambda$, $c_c = k_c \lambda$, $k_l = k_c$, $k_r = k_c$

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Scheme of the system

Scheme of the real object is presented in the figure. It was obtained by the analysis of real mechanism.



Analysis of the scheme allows to claim its number of degrees of freedom which is 2

Table with parameter values for calculations

The values of individual parameters adopted for the calculations are presented in the table 1

Tabela 1: Default parameter values

| Parameter | Value |
|-----------|---------------|
| m_1 | m |
| m_2 | m |
| c_r | $k_c \lambda$ |
| c_l | $k_c \lambda$ |
| c_c | $k_c \lambda$ |
| k_l | k_c |
| k_r | k_c |

Kinetic energy

Kinetic energy of the system has a following form:

$$T = 2m\dot{x}_l^2 + 2m\dot{x}_r^2 \quad (1)$$

Determined formula specify energy of the system related to its inertial properties.

Potential energy

Potential energy of the system has a following form:

$$V = k_c(-x_l + x_r)^2 + k_c x_l^2 + k_c x_r^2 \quad (2)$$

The presented relationship describes the interaction of potential force fields in which the object is located.

Dissipative Rayleigh function

The energy dissipated by attenuation is given by the formula:

$$D = \frac{k_c \lambda (\dot{x}_l - \dot{x}_r)^2}{2} + \frac{k_c \lambda \dot{x}_l^2}{2} + \frac{k_c \lambda \dot{x}_r^2}{2} \quad (3)$$

The given dependence is the Rayleigh dissipation potential, which when differentiated against the generalized velocity vector allows to determine the viscous damping forces.

Lagrangian of the system (Lagranges function)

System Lagrangian is described by the formula (4):

$$L = -2k_c x_l^2 - 2k_c x_r^2 + 2m \dot{x}_l^2 + 2m \dot{x}_r^2 + 2k_c x_l x_r \quad (4)$$

The Euler-Lagrange equations for the case under consideration are as follows:

$$\frac{d}{dt} \frac{\partial L}{\partial \dot{x}_l} + \frac{\partial D}{\partial \dot{x}_l} - \frac{\partial L}{\partial x_l} = Q_{x_l}^N \quad (5)$$

$$\frac{d}{dt} \frac{\partial L}{\partial \dot{x}_r} + \frac{\partial D}{\partial \dot{x}_r} - \frac{\partial L}{\partial x_r} = Q_{x_r}^N \quad (6)$$

Subsequent derivatives obtained with the Euler-Lagrange equations are as follows:

$$\frac{\partial L}{\partial x_l} = -4k_c x_l + 2k_c x_r \quad (7)$$

$$\frac{\partial L}{\partial x_r} = -4k_c x_r + 2k_c x_l \quad (8)$$

$$\frac{\partial L}{\partial \dot{x}_l} = 4m \dot{x}_l \quad (9)$$

$$\frac{\partial L}{\partial \dot{x}_r} = 4m \dot{x}_r \quad (10)$$

$$\frac{d}{dt} \frac{\partial L}{\partial \dot{x}_l} = 4m \ddot{x}_l \quad (11)$$

$$\frac{d}{dt} \frac{\partial L}{\partial \dot{x}_r} = 4m \ddot{x}_r \quad (12)$$

$$\frac{\partial D}{\partial \dot{x}_l} = -k_c \lambda \dot{x}_r + 2k_c \lambda \dot{x}_l \quad (13)$$

$$\frac{\partial D}{\partial \dot{x}_r} = -k_c \lambda \dot{x}_l + 2k_c \lambda \dot{x}_r \quad (14)$$

The results of the presented operations are used to determine the equations of motion of the system.

Equation of motion

Using the calculated derivatives, the equations of motions are based on the appropriate formulas. System equations of motions are described by the formulas (15)- (16)

$$F - F \sin(\Omega t) - 2k_c x_r + 4k_c x_l + 4m \ddot{x}_l - k_c \lambda \dot{x}_r + 2k_c \lambda \dot{x}_l = 0 \quad (15)$$

$$2k_c x_l + 4k_c x_r + 4m \ddot{x}_r - k_c \lambda \dot{x}_l + 2k_c \lambda \dot{x}_r = 0 \quad (16)$$

The determined equations constitute a mathematical dynamic description of the properties of the system. Further analysis allows for an effective analysis of the modeled object's operation and determination of its mechanical parameters.

Determining fundamental matrix

The matrix of masses and stiffnesses of the system was determined from the equations of motion:

$$M = \begin{bmatrix} 4m & 0 \\ 0 & 4m \end{bmatrix} \quad (17)$$

$$K = \begin{bmatrix} 4k_c & -2k_c \\ -2k_c & 4k_c \end{bmatrix} \quad (18)$$

The fundamental matrix, on the basis of which the characteristic equation of the considered system Δ was determined, is as follows:

$$A = \begin{bmatrix} 2ik_c\lambda\omega + 4k_c - 4m\omega^2 & -ik_c\lambda\omega - 2k_c \\ -ik_c\lambda\omega - 2k_c & 2ik_c\lambda\omega + 4k_c - 4m\omega^2 \end{bmatrix} \quad (19)$$

$$\Delta = 12k_c^2 + 16m^2\omega^4 - 32k_cm\omega^2 - 3k_c^2\lambda^2\omega^2 + 12ik_c^2\lambda\omega - 16ik_c\lambda m\omega^3 \quad (20)$$

The fundamental matrix allows you to define a fixed solution. On the other hand, based on the characteristic equation, the eigenfrequencies of the system are determined.

General solution

General solution is presented by expression:

$$\begin{aligned} X_{g-xl}(t) = & C_1 e^{-\frac{k_c\lambda t}{8m}} \cos\left(t\sqrt{-\frac{k_c^2\lambda^2}{64m^2} + \frac{k_c}{2m}}\right) + C_2 e^{-\frac{k_c\lambda t}{8m}} \sin\left(t\sqrt{-\frac{k_c^2\lambda^2}{64m^2} + \frac{k_c}{2m}}\right) \\ & + C_3 e^{-\frac{3k_c\lambda t}{8m}} \cos\left(t\sqrt{-\frac{9k_c^2\lambda^2}{64m^2} + \frac{3k_c}{2m}}\right) \\ & + C_4 e^{-\frac{3k_c\lambda t}{8m}} \sin\left(t\sqrt{-\frac{9k_c^2\lambda^2}{64m^2} + \frac{3k_c}{2m}}\right) \end{aligned} \quad (21)$$

$$\begin{aligned} X_{g-xr}(t) = & C_1 e^{-\frac{k_c\lambda t}{8m}} \cos\left(t\sqrt{-\frac{k_c^2\lambda^2}{64m^2} + \frac{k_c}{2m}}\right) + C_2 e^{-\frac{k_c\lambda t}{8m}} \sin\left(t\sqrt{-\frac{k_c^2\lambda^2}{64m^2} + \frac{k_c}{2m}}\right) \\ & - C_3 e^{-\frac{3k_c\lambda t}{8m}} \cos\left(t\sqrt{-\frac{9k_c^2\lambda^2}{64m^2} + \frac{3k_c}{2m}}\right) \\ & - C_4 e^{-\frac{3k_c\lambda t}{8m}} \sin\left(t\sqrt{-\frac{9k_c^2\lambda^2}{64m^2} + \frac{3k_c}{2m}}\right) \end{aligned} \quad (22)$$

General solution describes motion of the analysed system - presents displacement i function of time - and is given by considerations about free vibrations of the system

Steady solution

The steady solution is given by the formula:

$$\begin{aligned}
 X_{s-xl(t)} = & \left(\frac{0.125F \left(-\Omega^2 + \frac{0.5k_c}{m}\right)}{m \left(\frac{0.0625\Omega^2 k_c^2 \lambda^2}{m^2} + \left(-\Omega^2 + \frac{0.5k_c}{m}\right)^2\right)} + \frac{0.125F \left(-\Omega^2 + \frac{1.5k_c}{m}\right)}{m \left(\frac{0.563\Omega^2 k_c^2 \lambda^2}{m^2} + 2.25 \left(-0.667\Omega^2 + \frac{k_c}{m}\right)^2\right)} \right) \sin(\Omega t) \\
 & + \left(-\frac{0.333F}{m^2 \left(\frac{0.563\Omega^2 k_c^2 \lambda^2}{m^2}\right)} + \frac{0.333F}{k_c} \right)
 \end{aligned} \tag{23}$$

$$\begin{aligned}
 X_{s-xr(t)} = & \left(\frac{0.125F \left(-\Omega^2 + \frac{0.5k_c}{m}\right)}{m \left(\frac{0.0625\Omega^2 k_c^2 \lambda^2}{m^2} + \left(-\Omega^2 + \frac{0.5k_c}{m}\right)^2\right)} - \frac{0.125F \left(-\Omega^2 + \frac{1.5k_c}{m}\right)}{m \left(\frac{0.563\Omega^2 k_c^2 \lambda^2}{m^2} + 2.25 \left(-0.667\Omega^2 + \frac{k_c}{m}\right)^2\right)} \right) \sin(\Omega t) \\
 & + \left(\frac{0.167F}{m^2 \left(\frac{0.563\Omega^2 k_c^2 \lambda^2}{m^2}\right)} + \frac{0.167F}{k_c} \right)
 \end{aligned} \tag{24}$$

The specific solution is related to the presence of quantities that force motion (vibrations) of the analyzed system.